

Total No. of printed pages = 11

END SEMESTER EXAMINATION – 2021

Semester : 1st (New)

Subject Code : Sc-102

MATHEMATICS – I

Full Marks : 70

Time – Three Hours

The figures in the margin indicate full marks for the questions.

Instruction :

All questions of PART-A and PART-B are compulsory.

POLYTECHNIC DIPLOMA

PART – A

Marks – 25

1. Choose the correct answer of the following questions : $1 \times 10 = 10$

(i) The value of i^{109} is

(a) $-i$

(b) i

(c) -1

(d) 1

[Turn over

✓ (ii) The modulus of $a - ib$ is

(a) $a + ib$ (b) $\sqrt{a - ib}$

(c) $\sqrt{a^2 + b^2}$ (d) $\sqrt{a^2 - b^2}$

✓ (iii) If a polynomial $p(x)$ has no factor common to the factors of $x^3 - 1$, then the partial fraction of the proper fraction $\frac{p(x)}{x^3 - 1}$ be of the form

(a) $\frac{Ax^2 + Bx + C}{x^3 - 1}$

(b) $\frac{C}{x - 1} + \frac{A + Bx}{x^2 + x + 1}$

(c) $\frac{A}{x - 1} + \frac{Bx^2 + Cx + D}{x^2 + x + 1}$

(d) $\frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{C}{(x - 1)^3}$

(iv) If ${}^{n+1}C_3 = 2 \times {}^nC_2$, then the value of n is

★ (a) 3 (b) 4

(c) 5 (d) 6 ✓

✓ (v) How many arrangements can be made out of the letters of the word 'POLYTECHNIC' ?

- (a) $11!$ (b) $\frac{11!}{2!}$ ✓
(c) $11! - 2!$ (d) ${}^{11}P_2$

(vi) Number of terms in $(2-x^2)^{15}$ is

- ✗ (a) 16 (b) 15
(c) 14 ✓ (d) 30

(vii) Coefficient of middle term in expansion of

$$\left(x - \frac{x^3}{5}\right)^8 \text{ is}$$

- ✗ ✓ (a) $\frac{14}{625}$ (b) $\frac{70}{625}$
(c) $\frac{14}{125}$ (d) $\frac{70}{125}$

(viii) The value of $\log_{81} 27$ is

- ✓ (a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{5}{4}$ (d) $\frac{3}{4}$

(ix) If A is a matrix of order $m \times n$ and B is a matrix of order $n \times p$, then the order of AB is

- (a) $p \times m$ (b) $p \times n$
(c) $n \times p$ ✓(d) $m \times p$

(x) If A is a skew symmetric matrix of order 3×3 , then the sum of the diagonal elements of A is

- ✗ (a) 0 (b) 6
(c) 9 (d) 3

2. Choose the correct statement :

$$1 \times 5 = 5$$

(i) (a) $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$

✗ (b) $\sin(A+B)\sin(A-B) = \cos^2 A - \sin^2 B$

(c) $\sin(A+B)\sin(A-B) = \sin^2 A - \cos^2 B$

✓ (ii) (a) $\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$ ✓

$$(b) \sin 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$(c) \tan 2A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$(iii) (a) \sin 3\theta = 4 \sin^3 \theta - 3 \sin \theta \quad \checkmark$$

$$\times (b) \sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

$$(c) \sin 3\theta = 3 \sin^3 \theta - 4 \sin \theta$$

(iv) In any ΔABC

$$\checkmark (a) a^2 = b^2 + c^2 - 2bc \cos B$$

$$(b) a^2 = b^2 + c^2 + 2bc \cos A$$

$$(c) a = b \cos C + c \cos B \quad \checkmark$$

$$(v) (a) \sin^{-1}(x) + \sin^{-1}(-x) = \frac{\pi}{2}, x \in [-1, 1]$$

$$\checkmark (b) \cos^{-1}(x) + \cos^{-1}(-x) = \pi, x \in [-1, 1] \quad \checkmark$$

$$(c) \tan^{-1}(x) + \cot^{-1}(-x) = \frac{\pi}{2}, x \in \mathbb{R}$$

3. Fill in the blanks :

1×5=5

✓ (i) The volume of a sphere of radius 6 unit is 288π.

* (ii) The base radius of a cone is 7 units. If the height of the cone is 24 units, its lateral surface is . *4 unit*

* (iii) In the Simpson's Rule : $\text{Area} = \frac{S}{3}[A + 2D + 4E]$, the letter 'S' stands for .

* (iv) If m_1 and m_2 be the slopes of two perpendicular lines, then the relation between m_1 and m_2 is . *parallel*

(v) The equation of a straight line passing through (0,-1) and parallel to x-axis is .

4. Write true or false :

1×5=5

✓ (i) If two columns of a determinant are same (identical), then the value of the determinant will be zero. *✓*

* (ii) The value of $\sin(-1230^\circ)$ is $\frac{1}{2}$. *✓*

- * (iii) The principal value of $\cos^{-1}(\cos \frac{5\pi}{4})$ is $\frac{\pi}{4}$. \swarrow
- * (iv) In a triangle ABC, if the sides are $a=7, b=5$ and $c=8$, then the value of A will be 60° . \swarrow
- (v) If a base of field 55m and number of ordinates are 11, then the breadth of each strip is equal to 5m. \swarrow

PART - B

Marks - 45

5. Answer any *five* of the following questions :
 $2 \times 5 = 10$

- (i) If $x=1-i$, then prove that $x^2-2x+2=0$.
- (ii) Find the argument of $2+2\sqrt{-3}$.
- (iii) In how many different ways can the letters of the word PANDEMIC be arranged without changing the order of the vowels in the word ?
- (iv) Write the general term in the expansion of

$$\left(9x - \frac{1}{3\sqrt{x}}\right)^{18} \text{ and find its 13th term.}$$

(v) Prove that

$$\log 2 + 16 \log \frac{16}{15} + 12 \log \frac{25}{24} + 7 \log \frac{81}{80} = 1.$$

(vi) Insert three G.M.s between 1 and $\frac{1}{16}$.

(vii) If $A = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B^T = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A+2B)^T$.

6. Resolve into partial fractions (any one):

3×1=3

(i) $\frac{x}{(x+1)(x-2)^2}$

(ii) $\frac{2x-3}{x^2+6x+8}$

7. Answer any two of the following questions:

3×2=6

(i) If 3rd and 7th terms of an Arithmetic Progression are 18 and 30 respectively, then find the progression.

(ii) If a, b, c in A.P. and x, y, z are in G.P., then prove that $x^{b-c} y^{c-a} z^{a-b} = 1$.

(iii) By using properties of determinant find the value of

$$\begin{vmatrix} 1+x & y & z \\ x & 1+y & z \\ x & y & 1+z \end{vmatrix}$$

8. Prove any *three* of the following: $2 \times 3 = 6$

(i) $\frac{\cos 5^\circ + \sin 5^\circ}{\cos 5^\circ - \sin 5^\circ} = \tan 50^\circ$

(ii) $\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \sec \theta + \tan \theta$

(iii) $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = \frac{\pi}{4}$

(iv) In any triangle ABC, $\frac{a+b}{c} = \frac{\cos\left(\frac{A-B}{2}\right)}{\sin \frac{C}{2}}$

9. Answer any *two* of the following questions: $3 \times 2 = 6$

(i) If $A+B+C = \frac{\pi}{2}$, then prove that

$$\tan A \tan B + \tan B \tan C + \tan C \tan A = 1.$$

(ii) Solve : $2\cos^2 x + 3\cos x - 2 = 0$, $0 \leq x \leq 2\pi$.

(iii) Show that $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$.

10. Answer any *two* of the following questions :

$3 \times 2 = 6$

(i) An irregular plot has the following offsets measured from one end at equal distance.

X	0	12	24	36	48	60	72	84	96	108	120
d	53	52	47	49	53	63	58	61	52	49	48

Find the area of the plot.

(ii) Find the whole surface area of a right prism whose height is 75 cm and whose base is a regular octagon of side 12cm.

(iii) A regular pyramid has a base area of 56 cm^2 and a volume of 224 cm^3 . What is the height of the pyramid?

11. Answer the following questions : $2+3+3=8$

(i) If the three points $(3,0)$, $(a,-2)$ and $(8,2)$ are collinear, then find the value of a .

(ii) Find the equation of a straight line passing through $(-2, 4)$ and perpendicular to the line $4x + 5y + 2 = 0$.

(iii) Find the equations of two lines parallel to $5x - 12y + 26 = 0$ at a distance of 4 units from it.



POLYTECHNIC DIPLOMA